

Digital Calibration of RF Transceivers for I-Q Imbalances and Nonlinearity *

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Abstract

As Radio Frequency (RF) devices become more complex, the specifications become more stringent. In order to guarantee successful operation and compliance to certain specifications, digital correction techniques that compensate the device impairments are needed. In this paper, we present an analytical digital in-phase (I) and quadrature (Q) imbalance and non-linear compression correction methodology that improves the system bit error rate (BER). The gain and phase imbalances are corrected by using the gain and phase imbalance test data obtained during the product testing. The non-linear compression term is removed using Newton's method. The proposed test methodology is applicable for both burst based systems and continuous systems. Simulation results indicate that the proposed method improves the BER even under harsh noise contamination. The computational overhead of the compensation technique is minimal.

1 Introduction

The increasing demand for higher performance and faster data rates in wireless communications drives the industry for complex and extensively integrated systems [12]. Moreover, as the available bandwidth is decreasing, efficient utilization of the bandwidth through sophisticated modulation schemes is becoming essential [1]. These modulation schemes require that the RF subsystem be linear and the quadrature channels be perfectly matched. However, the inherent non-linear behavior of RF devices and the presence of process variations challenge these requirements. I-Q impairments as well as non-linear compression directly affect the quality of the device, which may even render the device in-operable, thereby reducing the yield. Fortunately, there is a potential to improve the overall yield through digital compensation techniques, which remove the detrimental effects

resulting from these impairments.

RF devices are extensively tested in order to guarantee their operation in the field under numerous worst case interference and blocking scenarios. While these test data is primarily used in order to classify devices as failure or acceptable, they can be potentially used in order to compensate for device impairments, such as quadrature gain and phase imbalances and non-linear compression. By correcting these impairments, the circuit can meet its high-level specifications such as Bit Error Rate (BER).

The calibration of quadrature imbalances is extensively studied in the literature as they allow devices that suffer from mismatched I and Q channels to be shipped out, thereby increasing the yield. In [14], the authors compute and store the I-Q imbalances during start-up in order to pre-distort incoming digital signals. The I-Q imbalances are predicted using a training signal and through loopback. Similarly, in [8], an on-chip phase-shift correction mechanism is employed. The capacitance of an on-chip varactor is controlled through a look-up-table to counteract the phase shift induced by the input signal.

While using on-circuit devices for I-Q imbalance compensation may be effective in order to meet the emission standards, there are numerous methods that target the I-Q mismatches through digital compensation techniques. In [13], a blind digital IQ estimation and compensation technique which employs the correlations between the I and Q channels is presented. As this method does not require any training data to predict the imbalance terms, it can be easily implemented. However, this method heavily relies on correlation computation, matrix multiplications, and eigenvalue decompositions, which may require extensive DSP resources. Similarly, in [2], the authors employ auto de-correlation to predict the I-Q mismatches. The number of multiplications is reduced by an adaptive algorithm, reducing the computation overhead. In [3], the authors employ a non-linear regression model to predict the mismatch parameters.

In addition to the high computational overhead, another significant shortcoming of the previously proposed techniques is that they only compensate for linear impairments.

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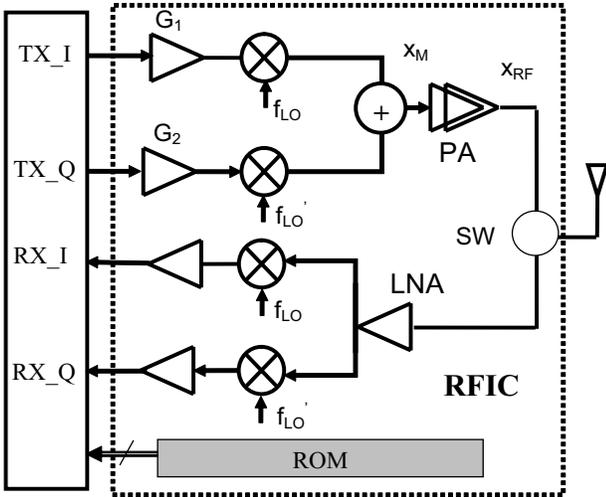


Figure 1. A Generic Quadrature Transceiver

As RF devices are inherently non-linear and the modern digital communication standards mandate the RF system to be extremely linear to perform as prescribed, these methods may be inadequate to improve the system quality.

In this paper we present an analytical constellation correction methodology by targeting gain and phase imbalances and non-linear compression. We correct signal impairments on received symbols individually without storing them into a memory for processing. Additionally, this method is also applicable to burst based communication systems. We first correct the gain and phase imbalances of the received symbols. We then use Newton's method to calculate and compensate for the non-linear compression term. Our method significantly improves the system BER and paves the way for increasing the overall yield.

2 Methodology

Quadrature transceivers, as illustrated in Figure 1, are built from orthogonal in-phase and out-of-phase channels and non-linear amplifiers that are hopefully operating in their linear operation range. A mismatch between these channels and the non-linear compression phenomenon will impact communicated symbols and may cause them to be erroneously interpreted. Figure 2 illustrates how quadrature imbalances combined with non-linear compression may degrade the system performance in the presence of additive white gaussian noise. Note that symbols do not deviate from their ideal locations uniformly, rather symbols with higher amplitudes tend to cluster outside their decision boundaries. This non-uniform behavior makes it challenging to compensate for non-linearity by utilizing existing compensation techniques.

When the amount of impairments on the system are

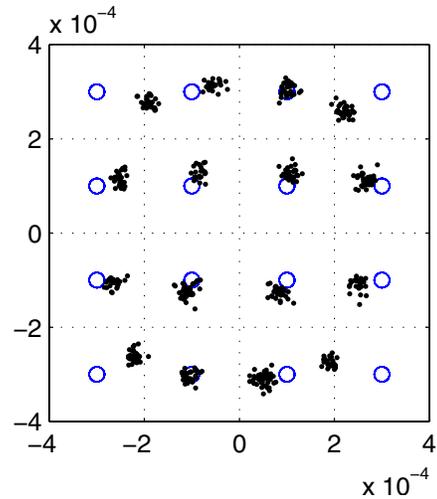


Figure 2. The dashed lines are the decision boundaries for symbols, circles are the ideal symbol locations.

known, they can be analytically calibrated by modeling quadrature imbalance and non-linear compression behaviors. The relevant test data that are obtained during manufacturing testing can be stored in a read only memory (ROM) unit to enable the digital signal processing (DSP) of the device to correctly identify received symbols. The ROM can be programmed easily during manufacturing and can be read during operation.

2.1 The Transceiver Model

Before presenting the details of the calibration methodology, we present the mathematical framework used to model the quadrature modulator by considering IQ imbalances and nonlinear behavior. The gain and phase imbalances can be attributed to the mismatches between the characteristics of the I and Q channels. While the I and Q channels may also possess non-linear characteristics, they are usually treated as linear since the extreme non-linear characteristics of the power amplifier forces designers to operate the rest of the system in the linear range [11][15]. Regardless of the underlying circuit implementations of each building block, the quadrature modulation device can be modeled with the inclusion of I-Q imbalances as follows:

$$x_M(t) = G\{I(t) \cos(\omega_c t) + (1+p)Q(t) \sin(\omega_c t + \phi)\}$$

$$G = G_I, \quad p = \frac{G_Q}{G_I} - 1, \quad \phi = \phi_Q - \phi_I \quad (1)$$

where ω_c is the carrier frequency, G_I and G_Q are the gains of I and Q channels, $I(t)$ and $Q(t)$ are the data signals, ϕ_I and ϕ_Q are the phases of the I and Q channels, G is the

common gain of the I and Q paths, p is the gain imbalance, and ϕ is the phase imbalance between I and Q paths. While this model encompasses all imbalance terms, the non-linear compression that originates from the RF front and back end needed to be modeled through a third order polynomial, as generally used in the literature [9],[10]:

$$x_{RF}(t) = \alpha_1 x_M(t) + \alpha_2 x_M(t)^2 + \alpha_3 x_M(t)^3 \quad (2)$$

where α_i is the i^{th} coefficient of the polynomial, and $x_{RF}(t)$ is the output of the power amplifier, as illustrated in Figure 1. Finally, we add zero-mean Gaussian white noise to the RF signal to obtain the final output signal, $x(t)$:

$$x(t) = x_{RF}(t) + n(t) \quad (3)$$

where $n(t)$ is a random signal to model additive white Gaussian noise that is generated in the system. Once all these impairments are imposed on the system, the I and Q symbols can be expressed as follows:

$$\begin{aligned} \hat{I}(t) &= (\alpha_1 + \Delta)[I(t) + (1+p)Q(t)\sin(\phi)] + \hat{n}(t) \\ \hat{Q}(t) &= (\alpha_1 + \Delta)(1+p)Q(t)\cos(\phi) + \hat{n}(t) \\ \Delta &= \frac{9}{16}\alpha_3\{I(t)^2 + [Q(t)(1+p)]^2 \\ &\quad + 2I(t)Q(t)(1+p)\sin(\phi)\} \end{aligned} \quad (4)$$

where $\hat{I}(t)$ is the received in-phase signal, $\hat{Q}(t)$ is the received quadrature signal, Δ is a non-linear compression term originating from the α_3 term, and $\hat{n}(t)$ is the received Gaussian white noise. By employing this equation, we can make several observations on the constellation diagram when there are several impairments, as illustrated in Figure 3. For instance, gain and phase imbalances only change the shape of the constellation diagram into a parallelogram. An interesting observation is that when there is also non-linear compression imposed on the quadrature imbalance system, the rotations of the parallelogram (γ_1 and γ_2) does not change. This behavior enables us to step by step correct the signal impairments. We first begin by compensating the gain and phase imbalances. We next compensate non-linear compressions using Newton's method.

2.2 Gain and Phase Imbalance Calibration

Since the gain and phase imbalances can be independently corrected using the non-linear compression term, we can eliminate the non-linear terms given in Equation 1 by taking $\Delta = 0$. By arranging terms and ignoring the random

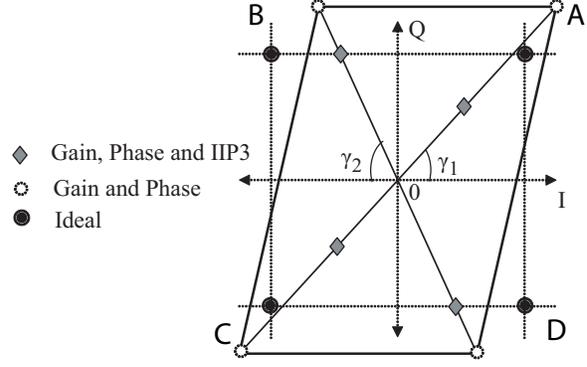


Figure 3. The Shape of the Constellation Diagram Under Certain Impairments

noise contribution, the compensated I and Q signals can be expressed as follows:

$$I' = \hat{I} - \frac{\sin(\phi)\hat{Q}}{\cos(\phi)} \quad (5)$$

$$Q' = \frac{\hat{Q}}{(1+p)\cos(\phi)} \quad (6)$$

where I' and Q' are the I and Q symbols which have already been corrected for gain and phase imbalance, \hat{I} and \hat{Q} are the received I and Q symbols, ϕ is the phase imbalance, and p is the gain imbalance term. Note that this equation utilizes the phase and gain imbalance coefficients that are obtained from the on-chip memory unit.

2.3 IIP₃ Calibration Using Newton's Method

While the non-linearity of the device creates unwanted spectral components, it also causes the amplitude of the signal at the fundamental frequency to compress. By comparing the compressions of the symbols at locations A and B in Figure 3, we observe that signals with higher powers are compressed more. This phenomenon indicates that unlike gain and phase imbalances, the non-linear compression discriminates between the symbols, causing high-power symbols to deviate more from their ideal locations. As the gain and phase imbalances are removed from the received symbol, we can re-arrange terms by setting gain and phase imbalances to zero:

$$\begin{aligned} \hat{I}'(t) &= (\alpha_1 + \Delta)I(t) + \hat{n}(t) \\ \hat{Q}'(t) &= (\alpha_1 + \Delta)Q(t) + \hat{n}(t) \\ \Delta &= \frac{9}{16}\alpha_3\{I(t)^2 + Q(t)^2\} \end{aligned} \quad (7)$$

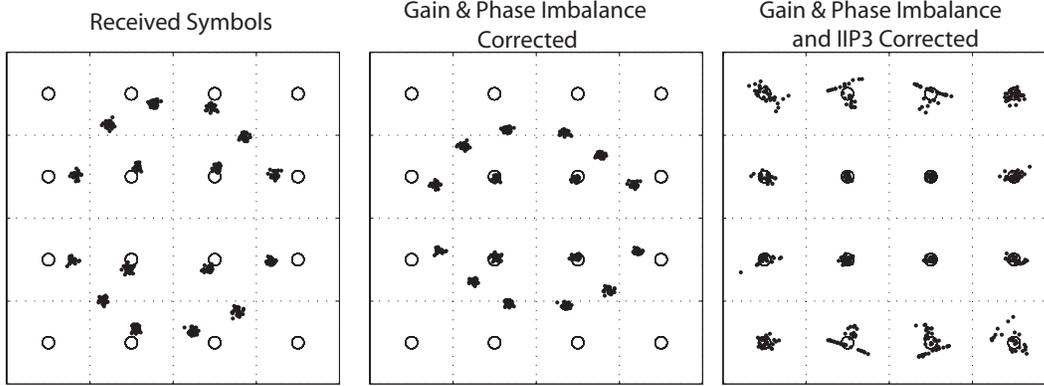


Figure 4. Impaired and Corrected Symbols

where I and Q are the resulting compensated signals, I' and Q' are the gain and phase imbalance corrected I and Q signals. The resulting system is a third order polynomial with two unknowns (I and Q) and with two known terms (I' and Q'). By re-arranging the terms, the non-linear system of equations can be formed as follows:

$$\begin{aligned}\hat{I}'(t) &= (\alpha_1 + \Delta)I(t) + \hat{n}(t) \\ \hat{Q}'(t) &= (\alpha_1 + \Delta)Q(t) + \hat{n}(t) \\ \Delta &= \frac{9}{16}\alpha_3\{I(t)^2 + Q(t)^2\}\end{aligned}\quad (8)$$

Instead of solving for two non-linear equations, we can form a new system of equations:

$$Q'(t) = \frac{-9}{16}(1 + \kappa^2)\alpha_3 Q^3(t) + \alpha_1 Q(t) \quad (9)$$

$$I(t) = \kappa Q(t) \quad (10)$$

$$\kappa = \frac{I'(t)}{Q'(t)} \quad (11)$$

where κ is the ratio of gain and phase imbalance compensated I and Q signals, α_1 and α_3 are the polynomial coefficients that are employed in the non-linear amplifier modeling. Since κ and $Q'(t)$ are known, we can solve for $Q(t)$ by employing a numerical root finding algorithm.

The Newton's method is an excellent candidate for this purpose since it converges rapidly. Furthermore, since the function is known and its derivative exists, the roots can be calculated accurately with a few iterations. The Newton's method requires an initial guess for the solution and converges to the solution by moving in the negative direction of the derivative at the initial guess. The Newton's iteration can be expressed as follows [4]:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad (12)$$

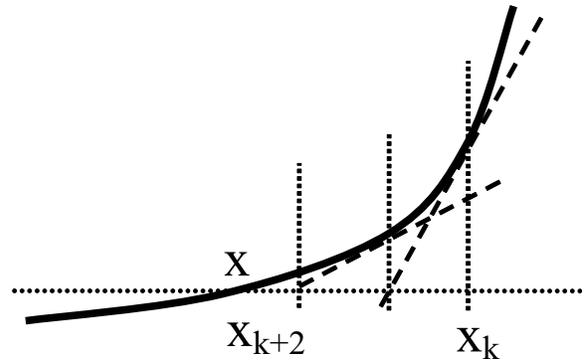


Figure 5. Newton's Method

where x_{k+1} is the next estimate for the solution calculated from the functions value at the x_k . Figure 5 illustrates the Newton's iterations. The derivative at the x_k location is utilized to guess the zero crossing (therefore the root). The initial guess (x_0) can be selected as the gain and phase imbalance compensated Q value ($Q'(t)$), as the solution will be close to it more than any other constellation point. The stopping criteria for iterations can be selected by considering the fact that the noise is also present in the signal and the computation time increases as the stopping criteria is stricter. In order to keep the computation cost acceptable, we stop the iterations if there is less than 0.1% difference between the last two solutions.

2.4 Noise

The transmitted and received signals are generally corrupted by numerous noise sources such as ambient noise and noise induced by analog circuits. Due to the random nature of the noise signal, received symbols cluster around a discrete location in the constellation diagram. As our calibration method works continually on the received sym-

Table 1. BER measurements under several impairments².

PI	GI	IIP ₃	BER	BER*	BER [†]	BER _{LB}
4.4	2.5	9.4	0.030	0.014	0.00033	0.000001
3.3	0.9	9.1	0.008	0.007	0.00024	0.000002
3.7	1.2	8.7	0.018	0.016	0.00048	0.000002
3.2	1.8	9.2	0.014	0.011	0.00032	0.000002
4.3	2.6	9.3	0.034	0.023	0.00093	0.000002
6.0	2.8	9.8	0.044	0.014	0.00039	0.000002
5.2	1.8	9.1	0.032	0.019	0.00067	0.000008
4.8	1.2	9.5	0.016	0.008	0.00050	0.000017
3.9	1.9	9.3	0.022	0.016	0.00083	0.000017
3.3	2.2	9.5	0.020	0.016	0.00085	0.000017
5.4	1.9	9.8	0.023	0.008	0.00040	0.000018
4.7	2.0	9.5	0.023	0.012	0.00055	0.000018
5.4	2.2	8.0	0.079	0.092	0.00610	0.000001

bols, we omit the noise terms from the equations and do not quantify the noise amount that may be present in the signal. Since noise is not averaged, it may also be present in the resulting symbols. In order to examine the robustness of our test methodology, we compute BER for noisy signals.

3 Simulation Results

In order to evaluate the robustness our technique, we construct an RF transceiver, as illustrated Fig 1, in MATLAB¹. We first apply our technique to impose several signal impairments, such as I-Q gain and phase imbalances, non-linearity, and noise to the digital baseband data. We then remove these imbalances sequentially to compute the BER.

As in many communication standards, we specify the acceptable system BER as 0.001 [7]. In order to measure the BER accurately, we apply a test signal that consists of 1 Million bits. We particularly focus on systems that have unacceptable BERs to show that our technique can bring the BER to within acceptable ranges. We inject noise, IQ gain and phase imbalances, and non-linear impairments randomly into the system components, as modeled in Equation 4. Table 1 shows the BER results for the RF transceiver under various impairment conditions. In this table, *PI* is the injected phase imbalance ($^{\circ}$), *GI* is the gain imbalance, *BER* is the uncorrected bit error rate, *BER** is effective BER after the gain and phase imbalances are corrected, and *BER[†]* is the effective system BER after all impairments are corrected. The last column in Table 1 shows the theo-

¹MATLAB[®] is a trademark of MathWorks Inc, <http://www.mathworks.com>

²PI: Phase imbalance ($^{\circ}$), GI: Gain imbalance, BER: Bit error rate without any calibration, BER*: Bit error rate with gain and phase imbalance calibration, BER[†]: Bit error rate with all imbalances are corrected.

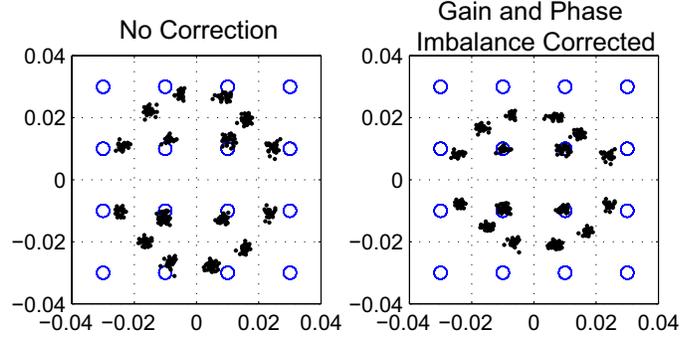


Figure 6. When only the I-Q imbalances are corrected, the BER may not improve. The uncompensated constellation diagram (left side) has a better BER than I-Q gain and phase imbalance corrected constellation diagram (right side).

retical BER based on the injected noise amount. This BER value corresponds to the theoretical lower-bound our correction methodology can reach. In all test cases, our proposed method reduces the effective system BER to within the specified limit of 0.001. Table 1 also shows that interestingly in some cases, the effective BER of the system increases as the I-Q imbalances are corrected. This odd behavior is actually due to the non-linearity of the system. Figure 6 illustrates the un-compensated and the I-Q imbalance corrected version of the constellation diagram for the first row in Table 1. Circled constellation points move closer to their decision boundary when the I-Q imbalances are removed. Since our method also compensates for the non-linear compression, the resulting BER of this system is well within the acceptable limits.

The computational overhead of the proposed method can be numerically determined through counting the number of floating point operations (FLOP). Table 2 show the FLOP counts for two different test set-ups with 1000 and 20000 bits. When there is no correction applied, the FLOP count is 194298. This number increases to 372870 when the proposed calibration technique is applied. This amounts to 0.9 times increase in the computation overhead. Furthermore, when the size of the input stimuli increases 20 times (20000 bits), the FLOP count also increases by 20 times, which in-

Table 2. The number of FLOPs with and without calibration

# Bits	No Correction	With Correction
1000	194298	372870
20000	3880298	7441192

icates that the proposed calibration technique is in the order of n ($O(n)$), where n is the input size. We can summarize the computational complexity of our proposed method as follows:

$$F_n = K \cdot O(n) \quad (13)$$

$$F_w = 1.9 \cdot K \cdot O(n) \quad (14)$$

where F_n is the FLOP count when there is no correction applied to the received symbols, F_w is the FLOP count when the proposed method is applied, K is a fixed constant, and $O(\cdot)$ is the order in big-O notation.

Based on the computational complexity of symbol determination, the computational overhead of the proposed method is 90%. Since the symbol determination is a minor computational effort compared to other signal processing functions, such as compression, coding, and decoding, the 90% increase compared to this baseline is not prohibitive.

Table 3. FLOP counts without IIP_3 calibration

Method	FLOPs
proposed	4263
non-data aided [6]	31864
data aided [5]	17006

Finally, Table 3 shows the FLOP count comparison with existing data-aided and non-data aided calibration methodologies [5],[6]. Since these methodologies calibrate only IQ gain and phase imbalances, we turn off our IIP_3 calibration for a fair comparison. All three methodologies operate on the same test signal, which consists of 500 bits and impaired with 0.1 gain imbalance and 15° phase imbalance. FLOP counts indicate that our method calibrates IQ imbalances with minimal number of floating point operations.

4 Conclusion

While the existing I-Q imbalance calibration methods decrease the bit error rate by compensating the mismatches, they may not significantly improve the BER for non-linear devices. In this paper we present a numeric I-Q calibration technique which employs the characterization data obtained during product testing in calibration.

Our method significantly improves the system BER even under non-linear impairments. The computational complexity of the proposed method is well within the acceptable limits.

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