IQ demodulation

$$S(t) = I(t) + iQ(t) = r(t)e^{i\varphi(t)} \ ,$$

i.e.

$$\mathrm{Re}\, S(t) = I(t) \ , \ \mathrm{Im}(t) = Q(t) \quad , \quad r(t) = |S(t)|^2 \ , \ \varphi(t) = \arg S(t) \ .$$

Diskrete *samples*, *sample rate* $1/T$: $\quad S_n = S(nT)$.
Sei f differenzierbar,

$$\dot{f}(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \quad .$$

Verschiedene Approximationen der Ableitung von $f_n = f(nT)$:

$$d_1 f_n = \frac{f_{n+1} - f_n}{T} \quad , \quad d_2 f_n = \frac{f_n - f_{n-1}}{T} \quad , \quad d_3 f_n = \frac{f_{n+1} - f_{n-1}}{2T}$$

FM Demod.

$$\varphi(t) = \arg S(t) = \arctan \frac{Q(t)}{I(t)} \quad ,$$

$$\begin{aligned} \dot{\varphi}(t) = \tfrac{d}{dt}\varphi(t) &= \frac{1}{1 + \left[\frac{Q(t)}{I(t)}\right]^2} \frac{d}{dt} \frac{Q(t)}{I(t)} = \frac{\dot{Q}(t)I(t) - Q(t)\dot{I}(t)}{I^2(t) + Q^2(t)} \\ &= \frac{\dot{Q}(t)I(t) - Q(t)\dot{I}(t)}{|S(t)|^2} \end{aligned} \qquad (*)$$

d1) Approximation $(*)$ linke Seite, $\dot{\varphi}(nT) \approx d_1\varphi_n = \frac{\varphi_{n+1} - \varphi_n}{T}$:

$$S_{n+1}\overline{S}_n = r_{n+1} r_n e^{i(\varphi_{n+1} - \varphi_n)} = I_{n+1}I_n + Q_{n+1}Q_n + i(Q_{n+1}I_n - I_{n+1}Q_n)$$

$$\begin{aligned} \rightsquigarrow \quad \varphi_{n+1} - \varphi_n = \arg(S_{n+1}\overline{S}_n) &= \arctan \frac{\mathrm{Im}(S_{n+1}\overline{S}_n)}{\mathrm{Re}(S_{n+1}\overline{S}_n)} \\ &= \arctan \frac{Q_{n+1}I_n - I_{n+1}Q_n}{I_{n+1}I_n + Q_{n+1}Q_n} \end{aligned}$$

d2) Approximation $(*)$ recht Seite, $\dot{Q}(nT) \approx d_1 Q_n$, $\dot{I}(nT) \approx d_1 I_n$:

$$\frac{(Q_{n+1} - Q_n)I_n - Q_n(I_{n+1} - I_n)}{|S_n|^2} = \frac{Q_{n+1}I_n - Q_n I_{n+1}}{|S_n|^2} = \frac{\mathrm{Im}(S_{n+1}\overline{S}_n)}{|S_n|^2}$$

d3) Wenn Signal FM-moduliert, dann $|S(t)| = \mathrm{const}$.